

AN ASYMMETRY COEFFICIENT FOR MULTIVARIATE DISTRIBUTIONS

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Abstract

The chiral index is an asymmetry coefficient for multivariate distributions. It measures the degree of indirect symmetry, not the degree of rotational symmetry. It is proportional to the Wasserstein distance between the distribution and its mirror image, minimized for all rotations and translations of the image, this minimised distance being normalized to the inertia of the distribution. The chiral index exists under the assumption of a finite and non null inertia. The chiral index is null if and only if the distribution is indirect-symmetric, i.e. achiral. In the one-dimensional case, the chiral index is very easy to compute on a pocket calculator, and it is proposed to be used as a symmetry test. It is an attractive alternative to Pearson's skewness based symmetry tests, because the Pearson's skewness is null for many asymmetric distributions and it assumes the existence of the third-order moment. Some other properties of the chiral index are also presented here.

References:

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Key Words: chiral index; Wasserstein distance; skewness; asymmetry coefficient; symmetry test